Further, $\varkappa$ is inversely related to the particle diameter; for example, a jet of velocity 3.43 m/sec showed a fall in $\varkappa$ from 0.35 to 0.15 as the equivalent diameter increased from 2.24 to 5.92 mm. Therefore, this coefficient is independent of the velocity but is dependent on the characteristics of the bed under these conditions.

NOTATION

b, b_d	are the concentration and dynamic radii of jet;
C, C_0, C_m, C_b	are the mass concentrations in the gas phase: current, initial, on the axis, and at the boundary;
d_0, d_e	are the packing diameter and equivalent diameter of solid particles;
r_0	is the radius of packing;
U, U_0, U_m, U_b	are the velocities: current, initial, on the axis, and at the boundary;
x	is the longitudinal coordinate;
y	is the transverse coordinate;
ε	is the porosity;
ρ_g	is the gas density;
$\varkappa$	is the experimental coefficient;
$\sigma = (C-C_b)/(C_m-C_b)$	is the dimensionless concentration;
$Z = (U-U_b)/(U_m-U_b)$	is the dimensionless velocity;
$\eta = y/b$	is the dimensionless transverse coordinate.

LITERATURE CITED

1. N. A. Shakhova and V. K. Lukashev, Abstracts of the "Thermiya-75" All-Union Conference [in Russian], Leningrad (1975).
2. N. A. Shakhova and V. K. Lukashev, Teor. Osn. Khim. Tekhnol., 9, No. 2 (1975).
3. G. N. Abramovich, Theory of Turbulent Jets [in Russian], Fizmatgiz, Moscow (1960).

ALLOWANCE FOR THE THERMAL BOUNDARY LAYER AND DIFFRACTION EFFECTS IN DETERMINING THE TRANSIT TIME OF SOUND IN ULTRASONIC FLOWMETERS

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The transit time of sound in ultrasonic flowmeters is investigated with allowance for the thermal boundary layer and diffraction effects.

In determining the time t_t in ultrasonic flowmeters, which is equal to the difference between the downstream and upstream transit times of sound, it is assumed that the temperature of the liquid is constant over the entire path from the source to the receiver. In real situations the liquid flowing in the duct often has a temperature other than that of the duct wall. In this case we know [1-3] that a thermal boundary layer is formed, in which there is a certain temperature distribution and outside of which the liquid temperature is roughly constant and equal to the temperature at the duct entry (Fig. 1). Under these conditions the velocity of sound propagation varies as a function of the temperature zone through which the sound wave passes. At

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a certain distance l_{te} from the duct entry (l_{te} is the length of the thermal entrant section) the temperature boundary layer fades away, whereupon the temperature distribution in the duct acquires a self-similar behavior and the temperature conditions of sound propagation differ from those in the thermal entrant section. The value of the sound velocity in liquids serves as a kind of "scale" for the determination of flow velocity with an ultrasonic flowmeter, but in many liquids it varies considerably with the temperature, and so the temperature conditions are directly related to the transit time t_t of a sound signal.

The theory of ultrasonic flowmeters deals with a plane wave. In this case the diffraction effects associated with downstream and upstream wave propagation, i.e., taking place in time t_t, are mutually compensating. In reality, however, because of the finite dimensions of the source and receiver the sound wave is not plane (it is well known – see [4], for example – that the field of the source has characteristic oscillations in the near zone, etc.), and the propagation of sound is no longer symmetrical in both directions. This fact also incurs error in the determination of t_t. Below we discuss the problem of evaluating t_t with allowance for these two factors.

We consider the problem of the thermal boundary layer. Directing our attention to non-Newtonian fluids, we investigate a liquid whose rheology is described by the Cross equation (we note that the solution of this problem for the Ellis equation is obtained by application of precisely the same procedure). We solve the problem under the following conditions: 1) the liquid flow is laminar and hydrodynamically stabilized; 2) the temperature of the duct wall at the site of the ultrasonic flowmeter is constant and equal to T_W; 3) the liquid is incompressible; 4) heat transfer by forced convection is much greater than heat transfer by conduction; 5) the temperature of the liquid at the duct entry is constant over the cross section and equal to T_0; 6) the rheodynamics and total heat-transfer process are constant; energy dissipation due to viscosity is negligible, and heat sources and sinks do not exist; 8) the influence of the thermal boundary layer and flow-velocity distribution over the duct cross section on the wave profile is negligible.

An exact solution of this problem is unobtainable. The approximate solution given below is obtained from a variational formulation of the problem based on the tenets of nonequilibrium thermodynamics [5-7]. The solution is sufficiently accurate for engineering computations (the error over the numerical method is 2 to 4%). Introducing the generalized dissipation function ψ, we have the following system of equations in cylindrical coordinates (the y axis coincides with the duct axis, and the origin with the center of the entry cross section; see Fig. 1):

$$\frac{\partial P}{\partial y}=\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{ry}),$$
$$\tau_{\dot{s}_i}=\mu_\infty+\left(\frac{\mu_0-\mu_\infty}{1+\alpha\dot{\gamma}^m}\right)^m, \tag{1}$$
$$\psi=\frac{1}{T^2}\left[\rho c_p U\frac{\partial T}{\partial t}\frac{\partial T}{\partial y}-k\frac{\partial^2 T}{\partial r^2}\frac{\partial T}{\partial t}+\frac{k}{2}\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial r}\right)^2\right],$$

subject to the boundary conditions $y = 0,\ 0 \le r \le R,\ T = T_0,\ y \ge 0,\ r = 0,\ \partial T/\partial r = 0;\ y > 0,\ r = R,\ T = T_W$.

The first two equations of the system (1) with the conditions $U = 0$, $\partial U/\partial r = 0$ at $r = 0$ are reducible to the form

$$\left[\mu_0+\alpha\mu_\infty(m+1)\left(\frac{dU}{dr}\right)^m\right]\frac{d^2U}{dr^2}-\frac{1}{2}\frac{dP}{dy}\left\{1+\alpha\left(\frac{dU}{dr}\right)^{m-1}\left[mr\frac{d^2U}{dr^2}+\frac{dU}{dr}\right]\right\}=0. \tag{2}$$

Solving (2) by the small-parameter method (restricted to the first approximation) and expressing the cross-section-mean velocity in terms of $\partial P/\partial y$, we obtain an expression for the velocity distribution:

$$U_s=2\bar{U}\frac{m+1}{m}\left[1-\left(\frac{r}{R}\right)^2\right]-\frac{\alpha(\mu_0-\mu_\infty)}{\mu_0R^m(m+2)}\left[\frac{4\bar{U}(m+1)}{m}\right]^{m+1}\left[1-\left(\frac{r}{R}\right)^{m+2}\right]. \tag{3}$$

It will be useful in what follows to transform to a Cartesian coordinate system x0z (Fig. 1). We approximate the temperature distribution by the polynomial [7]

$$T^*=\frac{T-T_0}{T_W-T_0}=1-\frac{10}{7}\frac{x}{\delta}+\frac{5}{7}\left(\frac{x}{\delta}\right)^4-\frac{2}{7}\left(\frac{x}{\delta}\right)^5. \tag{4}$$

Here $\delta(z)$ is the thickness of the thermal boundary layer. Using the fact that entropy production is minimal close to the steady state, we form the local potential from the last equation of the system (1):

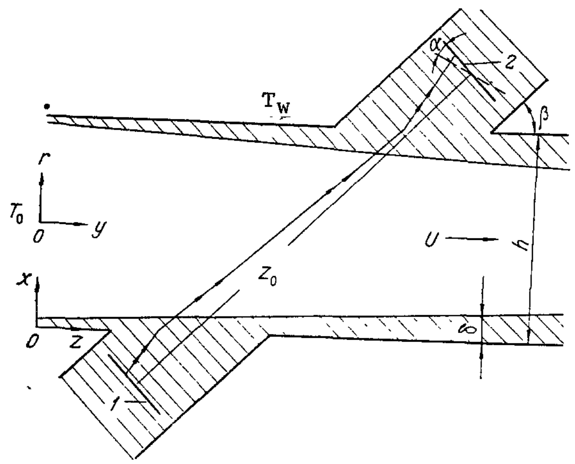

Fig. 1

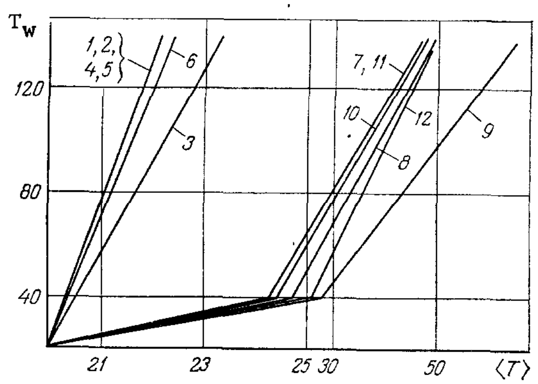

Fig. 2

Fig. 1. Diagram of sound propagation in an ultrasonic flowmeter with a thermal boundary layer. 1) Source plate; 2) receiving plate; δ) thickness of thermal boundary layer; T_0, T_W) temperature at duct entry and wall; α) angle of misalignment.

Fig. 2. Temperature T_W, °C, versus ⟨T⟩, °C, for various values of R, δ, and $\bar{U}$.

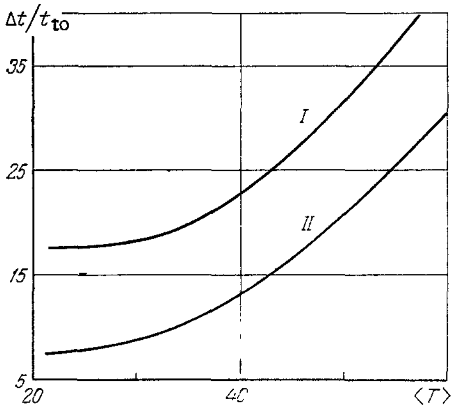

Fig. 3. Ratio $\Delta t/t_{t0}$ versus ⟨T⟩, °C. I) Calculated for R = 0.25 m; II) for R = 0.047 m.

$$E^* = \int_0^l \int_0^{\delta^0(z)} \left[\frac{k}{2} \left(\frac{\partial T}{\partial x} \right)^2 + \rho c_p U_s T \frac{\partial T^0}{\partial z} \right] dx dz. \tag{5}$$

We substitute (4) into (5) and vary E* with respect to δ in accordance with local potential theory. Then, setting δE* equal to zero and integrating with respect to z, we obtain for δ

$$c_1 \left(\frac{\delta}{R} \right)^3 + c_2 \left(\frac{\delta}{R} \right)^4 + c_3 \left(\frac{\delta}{R} \right)^5 = -\frac{k\Gamma z}{\rho c_p R^2}, \tag{6}$$

where the coefficients

$$c_1 = 0.77554\,(m+2)a_2, \quad c_2 = 0.13046 \left[(m+1)(m+2)\frac{a_2}{2} - a_1 \right],$$

$$c_3 = 0.08853\,(m+1)(m+2)\,a_2, \quad \Gamma = 0.29124,$$

$$a_1 = 2U\frac{m+1}{m}, \quad a_2 = \frac{\alpha(\mu_0 - \mu_\infty)}{\mu_0 R^m (m+2)} \left[\frac{4U(m+1)}{m} \right]^{m+1}.$$

From (6) we infer the following criterion for applicability of the solution obtained here:

$$\frac{2z\bar{U}}{R\,\mathrm{Pe}\,\{\bar{U}^m(m+2)\,a_2\,[0.88752+0.28508\,(m+1)]-0.13046a_1\}}\approx 1. \tag{7}$$

If we assume that the thickness of the thermal boundary layer grows linearly, for the determination of $\delta(z)$ we obtain in place of (6)

$$\delta=2z\bar{U}/(\mathrm{Pe}\,\{\bar{U}^m(m+2)\,a_2\,[0.88752+0.28508\,(m+1)]-0.13046a_1\}). \tag{8}$$

As mentioned, here we disregard possible distortions of the wave profile, so that it is useful to introduce the mean temperature

$$\langle T\rangle=\int_0^\delta U_s(x)\,T\,dx\Big/\int_0^l U_s(x)\,dx \tag{9}$$

and to investigate sound propagation in a liquid having the temperature $\langle T\rangle$. Calculating the latter according to (9), we obtain

$$\langle T\rangle=\frac{24\cdot 10^{-3}}{a_2\{(m+2)\,[12+(m+1)(m+4)]\}}\times \\ \times\left\{3.63m\,(m+1)(m+2)\,a_2\left(\frac{\delta}{R}\right)^4+32.823\left[\frac{(m+1)(m+2)}{2}a_2-a_1\right]\right. \\ \left.\times\left(\frac{\delta}{R}\right)^3+10.241\,[2a_1+(m+2)\,a_2]\left(\frac{\delta}{R}\right)^2+76.190a_2\frac{\delta}{R}\right\}. \tag{10}$$

Thus, the thermal boundary layer is accounted for by the introduction of $\langle T\rangle$, which is related to the average sound velocity $\langle c\rangle$.

We now consider the propagation of sound from the source to the receiver. We approximate the oscillations of the source by the oscillations of a piston radiator with a uniform distribution of the particle-velocity amplitude over its surface; we also assume that the pulsewidth is such that the receiver does not distort the field of the source. Finally, we assume that the receiver responds to the average pressure over its surface.

The problem of the source field and, in particular, of determining the time t_t can be solved on the basis of the Helmholtz equation and the boundary conditions for a piston radiator. If we use the Rayleigh form of the equation for the velocity potential, we obtain for the average pressure created on the receiving plate

$$\langle P\rangle=\mathrm{Re}\left(i\omega\rho e^{i\omega t}\frac{v_0}{2\pi}\int_{S_1}\int_{S_2}\frac{e^{-ikL}}{L}\,dS_1dS_2\right). \tag{11}$$

Equation (11) can be used in principal to solve numerically the problem of sound propagation in a moving medium. However, because of the enormous "machine time" required, the numerical integration of (11) is practically impossible. We have therefore integrated (11) approximately by means of the method of Lommel, certain readily deducible integral representations of the Bessel functions, and well-known relations from the theory of Bessel functions. Passing over the cumbersome integration procedure, we give the resulting expression for t_t with allowance for the thermal boundary layer and diffraction:

$$t_t=\frac{2r}{\langle c\rangle}\cos(\beta-\alpha_0)+\frac{1}{2\pi\nu}\left[\mathrm{Arctg}\left(\frac{M_2}{M_1}\right)_{U=-\bar{U}}-\mathrm{Arctg}\left(\frac{M_2}{M_1}\right)_{U=\bar{U}}\right], \tag{12}$$

where

$$M_1=L-E\sqrt{N_1^2+N_2^2}\cos(\psi-kE_1);\quad M_2=E\sqrt{N_1^2+N_2^2}\sin(\psi-kE_1);$$

$$E=z_0\langle c\rangle/k\pi\nu\,(ad_2\cos\alpha_0)^2;$$

$$E_1=\frac{a^2}{2z_0}\left(\cos\alpha_0+d_2+\frac{r\cos(\beta-\alpha_0)}{z_0}-\frac{a^2\cos^2\alpha_0}{2z_0}\right)+\frac{A_1^2}{8z_0\cos^2\alpha_0};$$

$$r=Ur_0\,\mathrm{sign}\,U/c;\quad r_0=h/\sin(\beta-\alpha_0);\quad A_1=2r\cos(\beta-\alpha_0);$$

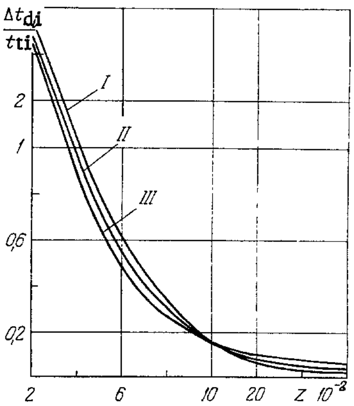

Fig. 4. Diffraction correction versus distance between acoustic plates. I) Average liquid temperature T = 20°C; II) 40°C; III) 60°C.

$$\psi = \operatorname{Arctg}\frac{N_2}{N_1}; \quad d_2 = 1 + r\cos(\beta - \alpha_0)/z_0;$$

$$\xi = ka^2 d_2 \cos\alpha_0 / z_0; \quad \eta = ka^2 T^0/2z_0;$$

$$N_1 = J_0(\eta)\sum_{n=0}^{\infty}(-1)^n(2n+1)\cos^{2(n-2)}\alpha_0 J_{2n+1}(\xi) + \left(\frac{A_1+2z_0\sin\alpha_0}{2ad_2\cos\alpha_0}\right)^i$$

$$\times\left[1 + \frac{A_1^2(A_1+2z_0\sin\alpha_0)^{-1}}{2ad_2\cos\alpha_0}\right]\sum_{i=1}^{\infty}\sum_{n=i+1}^{\infty}\sum_{n'=0}^{2(n-i)-2}\prod_{d=1}^{i}\left(\frac{n'+d}{d}\right)$$

$$\times(-1)^n\varepsilon\cos^{2(n-2)}\alpha_0 J_i(\eta) J_{2n-i-1}(\xi);$$

$$N_2 = J_0(\eta)\sum_{n=0}^{\infty}(-1)^n(2n+2)\cos^{2n-3}\alpha_0 J_{2n+2}(\xi) + \left(\frac{A_1+2z_0\sin\alpha_0}{2ad_2\cos\alpha_0}\right)^i$$

$$\times\left[1 + \frac{A_1^2(A_1+2z_0\sin\alpha_0)^{-1}}{2ad_2\cos\alpha_0}\right]\sum_{i=1}^{\infty}\sum_{n=i+1}^{\infty}\sum_{n'=0}^{2(n-i)-1}\prod_{d=1}^{i}\left(\frac{n'+d}{d}\right)$$

$$\times(-1)^n\varepsilon\cos^{2n-3}\alpha_0 J_i(\eta) J_{2n-i}(\xi);$$

$$L = J_0(ka\sin\alpha_0) + \frac{1}{2}\sum_{n=1}^{\infty}\frac{a^n}{(n+1)!}\left(\frac{k\sin\alpha_0}{2}\right)^n J_n(ka\sin\alpha_0);$$

$$T^0 = A_1 - 2z_0\sin\alpha_0; \quad k = 2\pi\nu/c; \quad \varepsilon = T^0/2ad_2\cos\alpha_0.$$

In Eq. (12) ⟨c⟩ is interpreted as the sound velocity corresponding to the average temperature ⟨T⟩ given by expressions (10), (6), and (8). The first term in (12) represents the difference between the downstream and upstream transit times of a plane wave, and the second term is the diffraction correction to that time. It was assumed in the derivation of (12) that $a^2/z_0^2 \ll 1$, $A_1^2/2ad_2 \ll 1$, as is almost always the case.

Figure 2 gives curves of ⟨T⟩ as a function of the duct wall temperature T_W, calculated according to Eq. (10). The temperature of the liquid at the duct entry is T_0 = 20°C. The constants have the following values: m = 0.7625; α = 0.0776 W/m°K; $\mu_0 - \mu_\infty$ = 0.161 N·sec·m². Curves 1-6 are calculated for δ = R/4, curves 1-3 for R = 0.047 m and $\bar{U}$ = 0.01, 0.1, 1.0 m/sec, respectively, and curves 4-6 for R = 0.2536 m and $\bar{U}$ = 0.01, 0.1, 1.0 m/sec; curves 7-12 are given for δ = 15R/16, i.e., close to the end of l_{te}, curves 7-9 for R = 0.047 m, and curves 10-12 for R = 0.2536 m. The velocities $\bar{U}$ have the same values in the same order as for curves 1-6. It follows from the curves of Fig. 2 that the distance from the duct entry to the site where ⟨T⟩ is determined is very crucial (among other factors affecting ⟨T⟩).

Figure 3 gives curves of $\Delta t/t_{t_0} = (t_{tT} - t_{t_0})/t_{t_0}$ in percent as a function of $\langle T \rangle$.

We can thus use Figs. 2 and 3 to find the error of determination of t_{tT} due to similarity of the temperatures of the liquid and duct wall. We see in Fig. 3 that the error of determination of t_{tT} can be very large and, for a fixed value of $\langle T \rangle$, increases with decreasing duct diameter. We note for comparison that for water with $T_0 = 20°C$, $T_W = 40°C$, $\bar{U} = 1$ m/sec, R = 0.25 m, $\delta = 15R/16$, and a parabolic velocity distribution in the duct cross section the error of determination of t_{tT} is 2.2%, while for $T_W = 120°C$ and identical values for all other parameters it is 6.1%. The foregoing results and discussion show that with the existence of a difference between T_W and T_0 the error of measurement of the time t_{tT} will depend, among other things, on the distance from the duct entry at which the instrument is placed.

Figure 4 gives the quantity $\Delta t_{di}/t_{ti}$ (i = 0, 1, and 2 at temperatures of 20, 40, and 60°C, respectively, for Δt_{di} and t_{ti}) as a function of the distance z_0 between the source and receiving plates. The calculations were carried out for the following values of the parameters $\nu = 1$ MHz; $a = 0.01$ m; $\bar{U}$ varied over the range from 0.01 to 9 m/sec; angle of inclination of acoustic channel relative to duct axis $\beta = 15°$; angle of misalignment of plates $\alpha_0 = 0°$. It is seen in the figure that curves II and III differ significantly from curve I only for small values of z_0. For z_0 of order 0.4 m the diffraction error $\Delta t_{di}/t_{ti}$ for all three curves becomes negligible (on the order of thousandths of one percent), i.e., diffraction effects are significant only for relatively short distances between the plates. We note also that the ratio $\Delta t_{di}/t_{ti}$ does not depend on the flow velocity in the investigated range.

NOTATION

τ	is the shear stress;
ρ	is the density of liquid;
c_p	is the specific heat at constant pressure;
k	is the thermal conductivity;
α	is the thermal diffusivity;
P	is the pressure;
Pe	is the Péclet number;
Re	is the real part of a number;
ω	is the cyclic frequency;
v_0	is the particle-velocity amplitude of sound source;
S_1, S_2	are the areas of source and receiving plates;
L	is the arbitrary distance between source and receiving plates;
μ_0, μ_∞	are the viscosities at zero and infinite shear velocities;
a	is the common radius of source and receiving plates;
m	is the rheological parameter;
$\dot{\gamma}$	is the shear velocity;
t_{t0}, t_{tT}	are the times equal to the difference between the downstream and upstream transit times of a plane wave at temperature $T_0 = 20°C$ and at a temperature $T = T_0$, respectively;
t_{ti} (i = 0, 1, 2)	is the time equal to the difference between the downstream and upstream transit times of a plane wave at temperature $T_0 = 20°$, $T_1 = 40°$, and $T_2 = 60°C$;
Δt_{di} (i = 0, 1, 2)	is the difference between diffraction corrections for downstream and upstream wave propagation at temperatures $T_0 = 20°$, $T_1 = 40°$, and $T_2 = 60°C$.

LITERATURE CITED

1. H. Schlichting, Boundary-Layer Theory, 6th ed., McGraw-Hill, New York (1968).
2. B. S. Petukhov, Heat Transfer and Drag in Laminary Fluid Flow [in Russian], Énergiya, Moscow (1967).
3. Heat and Mass Transfer in Non-Newtonian Fluids [in Russian], Énergiya, Moscow (1968).
4. M. B. Gitis and A. S. Khimunin, "Diffraction corrections for measurements of the absorption coefficient and velocity of sound," Akust. Zh., 14, No. 3 (1968).
5. I. Prigogine, Introduction to Thermodynamics of Irreversible Processes, Thomas, Springfield, Illinois (1955).
6. S. R. de Groot and P. Mazur, Non-Equilibrium Thermodynamics, Wiley-Interscience, New York (1964).
7. R. S. Schechter, The Variational Method in Engineering, McGraw-Hill, New York (1967).